**Lesson 0**

Infimum and Supremum

<https://www.cantorsparadise.com/why-we-use-the-infimum-and-supremum-in-mathematics-32a90ba13c6c>

The infimum and supremum are concepts in mathematical analysis that generalize the notions of minimum and maximum of finite sets

**Closed Intervals**

Consider the closed interval of

The minimum is very clearly and the maximum is very clearly

The points of the interval are represented by x where



**Open Intervals**

Consider the closed interval of

The open interval consists of all the numbers between and , but not and themselves

No minimum for the interval exists

No maximum for the interval exists



While minima and maxima may not exist for the open interval , we’d like to identify the next best alternatives.

We call a the ‘infimum’ of the open interval , and b the ‘supremum’.

The supremum of a set is its least upper bound. It is the least element in that is greater than or equal to each individual element of .

suprema (sup).

The infimum of a set is its greatest upper bound. It is the greatest element in that is less than or equal to each individual element of .

infima (inf)

can be expressed as x where , we see that a is the infimum

**Lesson 0**

Limit inferior and limit superior

**limit superior**

Also

**limit inferior**

Also

Example: ASS1 Q4 a i

find the limit (continuous) superior/inferior of , as

**Wolframalpha**

lim n -> inf (sqrt(2n+1)-sqrt(2n))

conjugate:

reciprocal:

power rule:

Example: ASS1 Q4 a ii

find the limit (continuous) superior/inferior of , as

**Wolframalpha**

lim n -> inf ((3n^3-n+8)/(4n(n-1)(n-2))

divide by leading term

product rule:

reciprocal:

power rule:

limiting value of sequence:

**Lesson 0**

Sequences

<https://tutorial.math.lamar.edu/Classes/CalcII/Sequences.aspx>

**General Sequence**

– first term

– second term

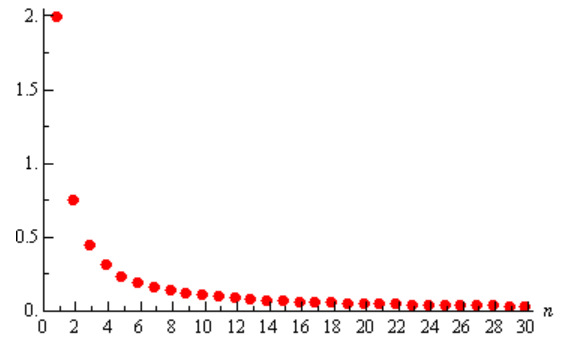
– term

– term

Denoted by:

Example: write down the first few terms of the following sequences

The graph:

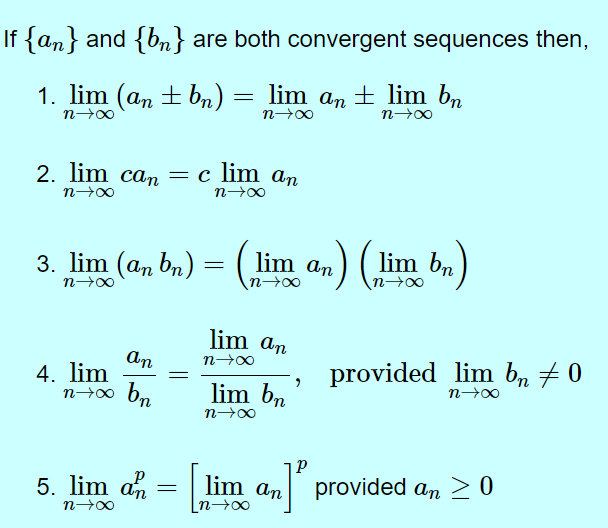


We then say that zero is the limit (or sometimes the limiting value) of the sequence.

limiting value of sequence:

**Convergent**

if exists and is finite



**Divergent**

if doesn’t exist or is infinite

example: determine if the following sequences converge or diverge

divide by leading term

the sequence converges and its limit is

**Lesson 0**

Monotone Sequences

<https://tutorial.math.lamar.edu/Classes/CalcII/MoreSequences.aspx>

Given the sequence

**Monotonic sequence**

Increasing sequence:

Decreasing sequence:

Lower bound (bounded below)

, for every

*There exists a number such that for every*

Upper bound (bounded above)

, for every

*There exists a number such that for every*

**Bounded Sequence**

If the sequence is both bounded below and bounded above

Example: Determine if the following sequences are monotonic and/or bounded

– first term for the first term

– second term for the second term

…

for every

the sequence is decreasing and monotonic.

Let

*Logically this is the only choice we can think of that fits*

Then for every

the sequence is bounded above

There are no numbers that exist that are lower than numbers in the sequence

the sequence is bounded above, but it is not bounded

Strictly Increasing

for all

Strictly Increasing

for all

**Lesson 0**

definition of the limit of a sequence

Definition: A real number is said to be a limit of a sequence if,

and only if

such that

for all

Example: ASS1 Q5 a.

Show by using an argument that

For the sequence

Let be given.

Let be given.

If , then

, then

such that

for all

Therefore,

**Lesson 0**

definition of continuity

<https://www.kent.ac.uk/smsas/personal/elm2/MA552/MA552Worksheet5.pdf>

Definition: Let and .

is continuous at if for every

there exists such that implies .

Example: Prove that is continuous at .

Let be given.

Let .

Then for we have

Since , and we have ,

so .

Therefore,

**Lesson 0**

Convergent Series

For a series to converge (or exist), then

If the value is any other real number other than or the limit does not exist, the series diverges.

Example: ASS2 2020 Q2a

In each of the following, and whether the given series is convergent or divergent and where appropriate conditionally or absolutely convergent.

Step 1: rewrite the equation in terms of

*Remember to write all in terms of*

Let

Then

Step 2: find the limit

**wolframalpha**

Limit[abs((-Power[3n,2]+2)/Power[4n,2])),n->∞]

*Remember brackets!!*

*Product rule*

*Evaluate limits as they tend to*

Step 3: if , the series diverges

if , the series converges

Since

Which implies

Then from the contrapositive of the vanishing condition the given series is divergent and no further testing is required.